Hyperspectral image fusion by the similarity measure based variational method

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Abstract

Hyperspectral remote sensing is widely used in many fields like agriculture, military detection, mineral exploration, and so on. Hyperspectral data has very high spectral resolution, but much lower spatial resolution than the data obtained by other types of sensors. The low spatial resolution restrains its wide applications. On the contrary, we easily obtain images with high spatial resolution but insufficient spectral resolution (like panchromatic images). Naturally, people expect to obtain images that have high spatial and spectral resolution at the same time by the hyperspectral image fusion. In this paper, a similarity measure based variational method is proposed to achieve the fusion process. The main idea is to transform the image fusion problem to an optimization problem based on the variational model. We first establish a fusion model that constrains the spatial and spectral information of the original data at the same time, then use the Split Bregman Iteration to obtain the final fused data. Also, we analyze the convergence of the method. The experiments on synthetic and real data show that the fusion method preserves the information of the original images efficiently, especially on the spectral information.

Keywords: Hyperspectral image fusion, Calculus of variations, Similarity measure, Split Bregman Iteration.

1 Introduction

Hyperspectral remote sensing is a new kind of remote sensing technology that attracts many people's attention recently. Using the imaging spectrometer, we obtain very high spectral resolution (usually less than 10 nm) three-dimension data cube. The main feature of the data cube is its combination of high spectral resolution and two-dimensional spatial image.[15,16,20] The image has almost continuous spectral curves which describes the reflectivity of the objects on the ground. The spectral information of the three-dimension data cube plays an important role in the image classification, detection and recognition. However, due to the constraint of bandwidth of the imaging spectrometer, the hyperspectral data has low spatial resolution. It restricts the application of hyperspectral remote sensing in some circumstance. On the other hand, we easily obtain high spatial resolution images such as panchromatic images, which have good spatial quality but insufficient spectral information. Naturally, how to get both high spatial and high spectral resolution images by fusing the two different kinds of former images is a crucial and useful task. The fusion process is the "hyperspectral image fusion" or "hyperspectral image sharpening" and the high spatial resolution image is the "master image".

One principle we have to obey in the remote sensing image fusion process is the effective combination of the spatial information of two different source images. It requires texture and details of the original images should be preserved in the fused images. Another is the preservation of spectral information, which means that the spectral information of the fused data should be close to that of the original hyperspectral data. Based on the principles, some methods were proposed to realize the fusion between the multispectral image (a kind of useful remote sensing image with less bands and worse spectral quality contrast to the hyperspectral image) and other kinds of images like the panchromatic image or the radar image. In fact, most existing fusion methods like PCA (Principal Component Analysis) method,[17] wavelet transformation method[7,12,22] and multiscale mathematical morphology method[11] are all proposed to realize that low-dimensional spectra image fusion. However, those methods are not completely applicable in hyperspectral image fusion because the hyperspectral image has more precise spectra and is more difficult to describe. If we directly use the above conventional methods to realize the hyperspectral image fusion, then the fused images are not ideal in spatial and spectral quality, especially in the preservation of spectra. Therefore, some researchers focus on other possible ways. An variational approach is one of them. This method can be traced to L. Rudin, S. Osher and E. Fatemi[13] in 1992 and it was used in many other image processing fields like image deblurring[3] and restoration.[14] Moeller et al.[10] extended Ballester's[1] work to hyperspectral image fusion.

In this paper, a new variational model for hyperspectral image fusion is proposed. We establish mathematical expressions which describe both the spatial and spectral information at the same time. The extremal function is just the fused image that we need. Therefore, the fusion process is transformed to be an optimization problem. This method has three advantages: No assumption on the spectral response of master image; No restraint on the number of hyperspectral data; Direct constraint on the spectra. Based on the former work, we obtain a similarity measure based variational model for hyperspectral image fusion and use the Split Bregman Iteration[6] to obtain the extremal function. Also we analyze the convergence of the iteration.

For hyperspectral image fusion, we should pay main attention to three key points:

1. The definition of functional space for functional expression. We have to describe the features of image like edges and texture based on the functional space.

2. The preservation of spatial and spectral information. We should preserve the spatial information of the master images while maintaining the spectral information of the hyperspectral images. The spatial information here represents the texture and the gray value.

3. The solution of the extremal function. Conventional iteration method converges relatively slow, so a faster iteration is needed.

The structure of the paper is as follows: In Section 2, we establish the model for the hyperspectral fusion problem, then the Split Bregman algorithm is used to solve the extremal function in Section 3, also we analyze the convergence of the algorithm. At last, we evaluate our method by the simulated and real data experiments in Section 4 and close with conclusions in Section 5.

2 Proposed variational model

In this section, the variational model will be established based on the previous principles. Three terms are need in all: First, the term E_g is used to describe the geometric information of the images. Second, the term E_s is used to preserve the spectra. Third, the term E_v is used to obtain the actual value of pixels. They describe the one aspect of the fused data respectively. Add them together and we get the final fusion model.

For the sake of convenience, we refer to the original hyperspectral image of band n as H_n, where n represents the number of spectrum. $P : \Omega \to R$ (where $\Omega \subset R^2$ is the image domain) represents the high spatial resolution image. r_n represents the fused image of band n. x and y correspondingly represents the horizontal and vertical direction.

2.1 Geometric function

The geometric function is designed to describe the geometric information (the geometric information of an image reflects its texture) contained in the original image. Ballester et al.[1] believed that the geometry of an image p can be represented by its level sets $X_\lambda = [p(x) \geq \lambda] = \{x : p(x) \geq \lambda\}$(resp. $X_\lambda^{'} = [p(x) \leq \lambda] = \{x : p(x) \leq \lambda\}$). The level sets of an image p represent its geometric information which we have to keep in the fused images. Therefore, how to describe the level sets of the image and keep the invariance is crucial to image fusion. To solve the problem, Ballester et al. used the vector field. It is defined as: $\theta(x) = \frac{\nabla P(x)}{|\nabla P(x)_\varepsilon|}$, where $|\nabla P|_\varepsilon = \sqrt{(D_x P)^2 + (D_y P)^2 + \varepsilon^2}$ is defined to avoid division by zero. Here, $D_x P$ and $D_y P$ are the partial differentiation of image P in x and y direction, respectively. Moeller et al. suggested to align the normal vectors of the level sets by using those of the master image.[9] Then the geometry of the fused image is close to that of the master image as a result and we have $|\nabla r_n| - \theta \cdot \nabla r_n = 0$ to every band of the fused image. Here, θ is the normal vector field of the master image. According to the paper[10] , we extend the first term of P+XS (short for Panchromatic and MultiSpectral images) model[1] to hyperspectral image fusion and obtain:

$$E_g = \sum_{n=1}^{N}[\alpha \int_{\Omega} |\nabla r_n| dx + \beta \int_{\Omega} div\theta \cdot r_n dx] \tag{1}$$

where α, β are both positive constants, N represents the number of bands and div is short for divergence. We can see that the first term of the function is the total variation which is defined in the bounded variational functional space. This space is widely used in image processing for its good description of some special image features such as edges.

2.2 Spectral function

This function is used to keep the spectral information of the hyperspectral data. As we have discussed before, one crucial aspect we have to consider in the hyperspectral image fusion is the preservation of spectra. This means we should make sure that the information of spectra contained in fused data is not distorted contrast to that of the original hyperspectral image. The spectral information includes the spectral angle mapper (SAM), location of extremal value, and so on. Moeller et al. chose the SAM as the invariance of the spectral information.[10] It is defined as SAM = $arccos(\frac{<\vec{a},\vec{b}>}{\|\vec{a}\| \cdot \|\vec{b}\|})$ where $\vec{a}, \vec{b}$ represents the the same pixel's spectral vector of the original and fused three-dimension data. Obviously, the SAM is expected to be zero for the two vectors could only be parallel in this condition. For hyperspectral image fusion, another equivalent form $r_i \cdot H_j - r_j \cdot H_i = 0$ is obtained at every pixel where H_i and H_j represent the original hyperspectral image of band i and j, and r_i and r_j represent the fusion hyperspectral image of band i and j, respectively. Therefore, we get the second function expression:

$$E_s = \gamma \sum_{i,j=1,i<j}^{N} \int_{\Omega} (r_i \cdot H_j - r_j \cdot H_i)^2 dx \tag{2}$$

where γ is also a positive constant.

2.3 Function of gray value

We have constrained the geometry and spectra of the data by the previous two functions. However, we need another expression to obtain the actual gray value of every pixel. Apparently, the fused images are expected to be close to both the original images (the hyperspectral image and the high spatial resolution image). In fact, we could get an image that is close to the two original images and use it to constrain the actual pixel value of the fused image. A modeling method based on similarity measure is proposed to get such an image Φ. By minimizing the distance between that image and the fused image, we obtain the final pixel value of the fused image. The pixel gray value function is written as follow:

$$D_n = \|r_n - \Phi_n(H_n, P)\|^2 \tag{3}$$

where H_n and P represent the original hyperspectral image of band n and the master image, respectively. r_n is the fused image. Φ_n represents the way that the two original images getting together. D_n means the distance between the original images and the fused image. The more the value is close to zero, the better. This model is more general since the multiple choices of Φ_n. Next, two different approaches to describe D_n are given, one is the morphology based similarity measure and another is the distance based similarity measure.

2.3.1 Morphology based similarity measure

To begin with, we introduce the multiscale morphology which has been widely used in image fusion.[5,8] Using the method, we easily obtain the bright and dark features and the basis of image by multiscale morphology opening and closing operations. First, different scales of morphology opening and closing operations are taken on the image and the differences between the adjacent scales are just the bright and dark features. The result obtained by the *max* scale operations is the basis shown in Fig. 1. Put the features and the basis together, we reconstruct an image. The gray value of image is good balance between the two original images. The reconstructed process is shown in Fig. 2, where H_n represents the low resolution image of band n and P is master image. Φ_n is what we need in (3).

Therefore, we get the similarity measure D_n by using the multiscale morphology. The third constraint function is proposed as follow:

$$E_v^{'} = \delta \sum_{n=1}^{N} \int_\Omega (r_n - \Phi_n)^2 dx \tag{4}$$

where δ is a constant. This is one way we constrain the actual value of fused image. Next, we will discuss another one.

2.3.2 Distance based similarity measure

As we have discussed before, the fused image r_n is required to be close to both two original images. To constrain the pixel gray value, we directly use the original two images as the Φ_n and obtain the the similarity measure $D_n = \|r_n - H_n\|^2 + \zeta\|r_n - P\|^2$. Therefore, another form of the constraint function is obtained as follow:

$$E_v^{''} = \eta \sum_{n=1}^{N} \int_\Omega (r_n - H_n)^2 + \zeta(r_n - P)^2 dx \tag{5}$$

where η is a constant, and ζ is the weight to balance the different effects of the hyperspectral image and the master image on the fused image.

From the previous discussion, we get four functions (1), (2), (4) and (5) that have their own specific meaning. Among them, (1) is used to constrain the geometric information of the fused image, (2) is used to realize the preservation of spectra, (4) or (5) is used to obtain the actual pixel gray value. Therefore, we obtain the functions to constrain the spatial and spectral information of the data, respectively. Put (1), (2) and (4) together and we obtain the final variational model which is named MSVM (Morphology based Similarity measure for Variational Model) in the following form:

$$\begin{aligned} E^{'} &= E_g + E_s + E_v^{'} \\ &= \sum_{n=1}^{N} [\alpha \int_\Omega |\nabla r_n| dx + \beta \int_\Omega div\theta \cdot r_n dx] \\ &\quad +\gamma \sum_{i,j=1,i<j}^{N} \int_\Omega (r_i \cdot H_j - r_j \cdot H_i)^2 dx \\ &\quad +\delta \sum_{n=1}^{N} \int_\Omega (r_n - \Phi_n)^2 dx \end{aligned} \tag{6}$$

On the other hand, put functions (1), (2) and (5) together and we get another variational model which is named DSVM (Distance based Similarity measure for Variational Model) :

$$\begin{aligned}
E^{''} &= E_g + E_s + E_v^{''} \\
&= \sum_{n=1}^{N}[\alpha \int_\Omega |\nabla r_n| dx + \beta \int_\Omega div\theta \cdot r_n dx] \\
&\quad +\gamma \sum_{i,j=1,i<j}^{N} \int_\Omega (r_i \cdot H_j - r_j \cdot H_i)^2 dx \\
&\quad +\eta \sum_{n=1}^{N} \int_\Omega (r_n - H_n)^2 + \zeta (r_n - P)^2 dx
\end{aligned} \tag{7}$$

If r_n makes $E^{'}$ (or $E^{''}$) be minimal, then it is the result that we need and that is the fused image of band n. Here, r_n is the "extremal function". In the next section, we will discuss how to solve the extremal function.

3 Algorithms for the variational models

Conventionally, we solve the extremal function by using the Euler-lagrange equation. However, an iteration algorithm named the Split Bregman Iteration is more efficient in the functional optimization.[6] Actually the related pioneer work on the Bregman iteration can be traced to Bregman's[2] work in 1967 and it was first used in image denoising.[6] In this section, we will solve the previous extremal function problem by using the Split Bregman Iteration, also we will analyze the convergence of this method.

3.1 The Split Bregman Iteration[6]

Considering a general optimization problem:

$$arg \min_{r} \quad |\phi(r)| + H(r) \tag{8}$$

where $arg \min$ represents the solution of extremal function. $\phi(r)$ and $H(r)$ are both convex and $H(r)$ is differential. We make such two substitutions $d = \phi(r)$ and $E(r,d) = |d| + H(r)$. Then we get the following optimization problem which is equivalent to (8):

$$arg\min_{r} \quad E(r,d) + \tfrac{\lambda}{2}\|d - \phi(r)\|^2 \tag{9}$$

where λ is a positive constant.

The main process using the Split Bregman Iteration is shown as follows:

$$p_r^{k+1} = p_r^k - \lambda(\nabla\phi)^T(\phi(r^{k+1}) - d^{k+1}) \tag{10}$$

$$p_d^{k+1} = p_d^k - \lambda(d^{k+1} - \phi(r^{k+1})) \tag{11}$$

where p_r^k and p_d^k represent the sub-differential of $E(r,d) + \frac{\lambda}{2}\|d - \phi(r)\|^2$ at r^k and d^k, respectively. The iteration (10) and (11) is equivalent to the following (12) and (13):

$$(r^{k+1}, d^{k+1}) = \min_{r,d} |d| + H(r) + \frac{\lambda}{2}\|d - \phi(r) - b^k\|^2 \tag{12}$$

$$b^{k+1} = b^k + (\phi(r^{k+1}) - d^{k+1}) \tag{13}$$

where b^k is a parameter that assists for the process of solving the extremal function. From the above iterations (10), (11), (12) and (13), we obtain the extremal function. For more details, please refer to the paper.[6, 14]

3.2 The application in the MSVM

For the fusion model (6) and (7), the Split Bregman Iteration could be used directly for their same form as the optimization problem (9). Now we apply this method to solve the extremal function of the MSVM model (6). It works as follows:

During the current iteration of r_n, if the value $\|r_n^{k+1} - r_n^k\|$ is larger than the threshold μ, the next three steps are performed:

1. Solve the equation:

$$\begin{aligned}&(2\delta I + 2\gamma \sum_{j=1, j\neq n}^{N} (H_j)^2) r_n^{k+1} - \lambda \Delta r_n^{k+1} \\ &= 2\eta\Phi_n - \beta div(\theta) + 2\gamma\Phi_n(\sum_{j=1, j\neq n}^{N} r_j^{k^*}\Phi_n) + \lambda div(d_n^k - b_n^k)\end{aligned} \tag{14}$$

where $\Delta = \nabla_{xy}^2$ and I is the identity matrix.

$$r_j^{k^*} = \begin{cases} r_j^{k+1} & \text{if } j < n \\ r_j^k & \text{if } j > n \end{cases} \tag{15}$$

By using the Gauss-Seidel algorithm, we get the r_n^{k+1} of (14) easily. Next, we update the variables d_n^k and b_n^k:

2. Update d_n^k:

$$(d_n^{k+1})_x = max(s^k - \frac{1}{\lambda}, 0)(\nabla_x r_n^{k+1} + (b_n^k)_x)/s^k \tag{16}$$

$$(d_n^{k+1})_y = max(s^k - \frac{1}{\lambda}, 0)(\nabla_y r_n^{k+1} + (b_n^k)_y)/s^k \tag{17}$$

where $(d_n^k)_x$ and $(d_n^k)_y$ represent the gradient of d_n^k in x direction and y direction, respectively. $(b_n^k)_x$ and $(b_n^k)_y$ represent the gradient of b_n^k in x direction and y direction, respectively. Here, $s^k = \sqrt{|\nabla_x r_n^{k+1} + (b_n^k)_x)|^2 + |\nabla_y r_n^{k+1} + (b_n^k)_y|^2}$.

3. Update b_n^k:

$$(b_n^{k+1})_x = (b_n^k)_x + (\nabla_x r_n^{k+1} - (d_n^{k+1})_x) \tag{18}$$

$$(b_n^{k+1})_y = (b_n^k)_y + (\nabla_y r_n^{k+1} - (d_n^{k+1})_y) \tag{19}$$

By repeating the three steps until $\|r_n^{k+1} - r_n^k\| < \mu$ is satisfied, we get the extremal function r_n which is just the fused image of band n.

Therefore, we have the whole process of solving the MSVM by using the Split Bregman Iteration:

For the band n

Initialize: $r_n^0 = H_n, (d_n^k)_x = 0, (d_n^k)_y = 0, (b_n^k)_x = 0, (b_n^k)_y = 0$

While $\|r_n^{k+1} - r_n^k\| > \mu$

solve the following equation to obtain r_n^{k+1}:

$$(2\delta I + 2\gamma \sum_{j=1, j\neq n}^{N} (H_j)^2) r_n^{k+1} - \lambda \Delta r_n^{k+1}$$
$$= 2\eta\Phi_n - \beta div(\theta) + 2\gamma\Phi_n(\sum_{j=1, j\neq n}^{N} r_j^{k^*}\Phi_n) + \lambda div(d_n^k - b_n^k)$$

update d_n^k and b_n^k :

$$(d_n^{k+1})_x = max(s^k - \frac{1}{\lambda}, 0)(\nabla_x r_n^{k+1} + (b_n^k)_x)/s^k$$
$$(d_n^{k+1})_y = max(s^k - \frac{1}{\lambda}, 0)(\nabla_y r_n^{k+1} + (b_n^k)_y)/s^k$$
$$(b_n^{k+1})_x = (b_n^k)_x + (\nabla_x r_n^{k+1} - (d_n^{k+1})_x)$$
$$(b_n^{k+1})_y = (b_n^k)_y + (\nabla_y r_n^{k+1} - (d_n^{k+1})_y)$$

3.3 The application in the DSVM

The application in the DSVM is similar to the MSVM except for the step (14). In this step, we have:

$$(2\eta I + 2\eta\zeta I + 2\gamma \sum_{j=1, j\neq n}^{N} (H_j)^2) r_n^{k+1} - \lambda \Delta r_n^{k+1}$$
$$= 2\eta H_n + 2\eta\zeta P - \beta div(\theta) + 2\gamma H_n (\sum_{j=1, j\neq n}^{N} r_j^{k^*} H_n) + \lambda div(d_n^k - b_n^k) \quad (20)$$

Therefore, the whole process of solving the DSVM by the Split Bregman Iteration is as follows:

For the band n

Initialize: $r_n^0 = H_n, (d_n^k)_x = 0, (d_n^k)_y = 0, (b_n^k)_x = 0, (b_n^k)_y = 0$

While $\|r_n^{k+1} - r_n^k\| > \mu$

solve the equation to obtain r_n^{k+1}:

$$(2\eta I + 2\eta\zeta I + 2\gamma \sum_{j=1, j\neq n}^{N} (H_j)^2) r_n^{k+1} - \lambda \Delta r_n^{k+1}$$
$$= 2\eta H_n + 2\eta\zeta P - \beta div(\theta) + 2\gamma H_n (\sum_{j=1, j\neq n}^{N} r_j^{k^*} H_n) + \lambda div(d_n^k - b_n^k)$$

update d_n^k and b_n^k :

$$(d_n^{k+1})_x = max(s^k - \frac{1}{\lambda}, 0)(\nabla_x r_n^{k+1} + (b_n^k)_x)/s^k$$
$$(d_n^{k+1})_y = max(s^k - \frac{1}{\lambda}, 0)(\nabla_y r_n^{k+1} + (b_n^k)_y)/s^k$$
$$(b_n^{k+1})_x = (b_n^k)_x + (\nabla_x r_n^{k+1} - (d_n^{k+1})_x)$$
$$(b_n^{k+1})_y = (b_n^k)_y + (\nabla_y r_n^{k+1} - (d_n^{k+1})_y)$$

3.4 Convergence analysis

Here we get a conclusion: If the expressions (1), (2) and (4) (or (5)) are all convex, then the algorithm we proposed converges. A more detailed analysis is given:

In the paper,[21] a similar work was done on the Bregman iteration. Here, we want to analyze how the Split Bregman iteration behaves in hyperspectral image fusion.

As we see, the key step in the process of the Split Bregman Iteration is (12) and (13), which are equivalent to (10) and (11). Substitute the r_n in the fusion model MSVM (6). Then (10) and (11) become:

$$p_{nr}^{k+1} = p_{nr}^k - \lambda(\nabla\phi)^T(\phi(r_n^{k+1}) - d_n^{k+1}) \quad (21)$$
$$p_{nd}^{k+1} = p_{nd}^k - \lambda(d_n^{k+1} - \phi(r_n^{k+1})) \quad (22)$$

The main analysis is focused on the d_n^k and r_n^k. Next, we should consider how they behave in the fusion process. To begin with, we rewrite the d_n:

$$|d_n|_\epsilon = \sum_i \sqrt{{d_{ni}}^2 + \epsilon} \quad (23)$$

$|d_n|_\epsilon$ is convex. To the hyperspectral image fusion model, we have:

$$\phi(r_n) = \nabla r_n \quad (24)$$

The expressions (2) and (4) are used to constrain the spectra and pixel gray value, respectively. We put them together and rewrite as follows:

$$H(r_n) = \kappa \sum_{n=1}^{N} \int_\Omega (r_n - T)^2 dx \quad (25)$$

where κ and T are both constants that are relative to (2) and (4). r_n^k is regarded as a constant in (22) and p_d^{k+1} and p_d^k are the sub-differential of $E(r_n, d_n) = |d_n| + H(r_n)$ at d_n^{k+1} and d_n^k by using the mean value theorem, we have

$$p_{nd}^{k+1} - p_{nd}^k = D_{n(i,i)}^{k+\frac{1}{2}}(d_n^{k+1} - d_n^k) \tag{26}$$

where

$$D_{n(i,i)}^{k+\frac{1}{2}} = \epsilon((d_{ni}^{k+\frac{1}{2}})^2 + \epsilon)^{-\frac{3}{2}} \tag{27}$$

$d_n^{k+\frac{1}{2}}$ is between d_n^{k+1} and d_n^k. $D_n^{k+\frac{1}{2}}$ is a diagonal matrix. Make substitution $Q^{-1} = D_n^{k+\frac{1}{2}}$ and $f = \phi(r_n^{k+1})$, put (22), (26) and (27) together, thus, we have:

$$(I + \lambda Q)(d_n^{k+1} - f) = d_n^k - f \tag{28}$$

In fact, we have $\lambda = 1$ in our fusion algorithms. Therefore, we rewrite (28) as follows:

$$(I + Q)(d_n^{k+1} - f) = d_n^k - f \tag{29}$$

From the equation (29), we see that, to the edges of image, $d_{ni}^{k+\frac{1}{2}}$ is much larger than ϵ, then Q is large, and d_n^k converge rapidly. Next we analyze (21). Here, d_n^k is regarded as a constant. Put three equations (21), (26) and (27) together, we get:

$$p_{nr}^{k+1} - p_{nr}^k = -(\nabla\phi)^T D_n^{k+\frac{1}{2}}(d_n^{k+1} - d_n^k) \tag{30}$$

where p_{nr}^{k+1} and p_{nr}^k are the sub-differential at r_n^{k+1} and r_n^k, respectively. $\phi(r_n)$ represents the gradient $\nabla(r_n)$. Again, using the mean value theorem, we get

$$A_n(r_n^{k+1} - r_n^k) = -Q(\Delta)^T(d_n^{k+1} - d_n^k) \tag{31}$$

where A_n is a constant that is correlated to the differential of $H(r_n)$ and Q here is also regarded as a constant. As we have discussed before, d_n^k converges rapidly. That means $d_n^{k+1} - d_n^k$ converges to zero rapidly. In the equation (31), $(\Delta)^T(*)$ is operation to calculate the transposed matrix of the second derivative. Since $d_n^{k+1} - d_n^k$ converges to zero rapidly, r_n^k also converges rapidly. Therefore, we see that, the Split Bregman Iteration converges rapidly in the hyperspectral image fusion. Also we see that the analysis is applicable for the model DSVM (7), the main process of analysis is similar.

4 Numerical Experiments

4.1 Experiment 1

We obtain the hyperspectral image of urban scene with 100 bands from the internet[10, 23] free, then use it to simulate our initial data. First we choose the image of band 4 as the master image. Next we subsample (by 0.25) the hyperspectral data, interpolate it to the same size as before and take it as our original hyperspectral data. Thus, in the experiment, the resolution of the master image is as 4 times as that of the hyperspectral image. Both images are $307 * 306$ and registered. Fig. 3 shows the two kinds of images.

The parameters we choose for MSVM are $\alpha = 1, \beta = 0.5, \eta = 2, \gamma = 0.5, \lambda = 1,\ \mu = 0.4$ and parameters $\alpha = 1, \beta = 1, \eta = 0.2, \gamma = 1, \lambda = 1,\ \zeta = 2$ ($\zeta = 0.5$) for DSVM. The fused images have high spatial quality with $\zeta = 2$ and high spectral quality with $\zeta = 0.5$. We also experiment some other conventional fusion methods including the wavelet method[22] and the multiscale morphology method.[11] Also, comparisons on subjective vision and objective indices are done among these different methods.

Fig. 4 shows the final images in false color. Fig. 4(a) is the original hyperspectral image. Fig. 4(b) is the fused image obtained by the wavelet method, Fig. 4(c) is obtained by the multiscale morphology method, Fig. 4(d) is obtained by the MSVM method, Fig. 4(e) is the fused image with high spatial resolution obtained by the DSVM method, and Fig. 4(f) is the fused image with high spectral resolution obtained by the DSVM method.

Among all the images, the wavelet method gets a balanced spatial and spectral image in Fig. 4(b) and the multiscale morphology method seems to behave better in describing details of image (Fig. 4(c)). In Fig. 4(d), we see that the details obtained by the MSVM method are very good while the spectral quality is improved contrasts to that in Fig. 4(c). In Fig. 4(e) and Fig. 4(f), we get high spatial and spectral images by using the DSVM method, respectively. It is apparently that the color of image Fig. 4(f) is closest to the initial image Fig. 4(a). Therefore, this method is a good tool in different situation with various demands.

In order to analyze the different performance of previous methods quantitatively, we need some objective indices. Here, the following four typical indices are chosen:

1. Average Gradient (AG).

AG=$\frac{1}{(M-1)(N-1)}\sum_{i=1}^{M}\sum_{j=1}^{N}\sqrt{((\nabla x)^2+(\nabla y)^2)/2}$.

Where M and N is the width and height of image. ∇x and ∇y are the gradient of x direction and y direction, respectively. This index is used to reflect texture of image and describe the details. The larger it is, the image we get is better in it is spatial information.

2. Correction Coefficient (CC).

CC=$\frac{\sum_{x=1}^{M}\sum_{y=1}^{N}[r(x,y)-\overline{r(x,y)}][H(x,y)-\overline{H(x,y)}]}{\sqrt{\sum_{x=1}^{M}\sum_{y=1}^{N}[r(x,y)-\overline{r(x,y)}]^2\sum_{x=1}^{M}\sum_{y=1}^{N}[H(x,y)-\overline{H(x,y)}]^2}}$.

Where $r(x,y)$ and $H(x,y)$ represent the pixel value of location (x,y) in two images. $\overline{r(x,y)}$ and $\overline{H(x,y)}$ represents the mean value of image r and H, respectively. The index CC evaluates the correlative degree of the fused image and the original images.[19] Here we used it to evaluate the correlation between the fused images and the original images.

3. Spectral Angle Mapper (SAM).

The definition is SAM $= arccos(\frac{<\vec{a},\vec{b}>}{\|\vec{a}\|\cdot\|\vec{b}\|})$. This is a very important index in describing the performance of spectral preservation. In our experiments, we calculate the SAM with all pixels and use their mean value as our final spectral angle.

4. Q-average (Q).

Q $= \frac{4\sigma_{r_n H_n}\overline{r_n}\overline{H_n}}{(\sigma_{r_n}^2+\sigma_{H_n}^2)[\overline{r_n}^2+\overline{H_n}^2]}$, where $\overline{r_n}$ and $\overline{H_n}$ represent the average value of images r_n and H_n respectively. σ_{r_n} and σ_{H_n} are the variances of images r_n and H_n, respectively.

This index reflects the combination of three different factors: "loss of correlation, luminance distortion, and contrast distortion".[18]

Table 1 shows the objective indices of the experiment 1. The bold font is the best one in the same row. In the experiment 1, the indices SAM and Q of the the DSVM method are the best of all. That means the DSVM method has advantage in preserving the spectra. The CC (hyperspectral image) means the correlation coefficient between the fused data and the hyperspectral data. We can see that the fused data is more close to the hyperspectral data than the master image. On the other hand, the MSVM method and the morphology method have advantages in the index AG contrast to other methods, which means they do better in describing the details of image. In a word, the similarity measure based fusion model has advantages in hyperspectral image fusion apparently.

As we have discussed before, the preservation of spectra is a crucial aspect in hyperspectral image fusion process. In order to deeply understand how the methods behave in preserving the spectra, Fig. 5 show the actual spectral curves of original hyperspectral data and the fused data obtained by the different methods. Fig. 5(a)-(e) is the spectral curve of the pixel (100,100) in the original image and the fused image obtained by the wavelet method, the morphology method, the MSVM method and the DSVM method, respectively. In all the images, the red curves are from the original data while the blue curves are from the fused data.

It is obviously that, contrast to the other methods, the DSVM keeps the spectra best of all (Fig. 5(d)). Though the MSVM (Fig. 5(c)) doesn't behave as well as the DSVM, it has higher spatial quality than the DSVM and higher spectral quality than the morphology method (Fig. 5(b)). The results are correspond to the objective indices in table 1.

In order to explain the rationality we directly add the three terms $E_g, E_s, E_v^{'}$ in model (6) and E_g, E_s and $E_v^{''}$ in model (7), here, we give the value of ratios between those three terms $E_g, E_s, E_v^{'}$($E_v^{''}$) and E_g .

In table 2, the value in the second row are the ratios between $E_g, E_s, E_v^{'}$ and E_g in MSVM and the value in the fourth row are the ratios in DSVM. We can clearly see that the maximal ratio is 4.293 while the minimum is 0.433. So the terms E_g, E_s and $E_v^{'}(E_v^{''})$ have similar range and it is reasonable that we add the terms $E_g, E_s, E_v^{'}$ ($E_v^{''}$) together in equation (6) and (7).

4.2 Experiment 2

The hyperspectral image we use in experiment 2 is the Tuosu Lake in Qinghai Province of China with 100 bands.[24] Its spatial resolution is 100m. Fig. 6(a) shows the band 4 of hyperspectral image. The master image of the same scene is obtained from the Google maps[25] with the spatial resolution of 30m. They are manually registered before the experiment.

Also, we experiment on the data by using the different methods and the fused images are all shown in false color in Fig. 7.

From Fig. 7(a), we can see that the wavelet method get a good result in spatial quality for most details of the master image are preserved. The result obtained by the morphology method in Fig. 7(b) seems to behave better in spatial quality because this method enhances the details of image. However, we clearly see that the color of the above two images are distorted contrast to Fig. 7(a). That means the above two methods are not dominant in describing the spectra. In Fig. 7(d), we also get a result with a good spatial quality. At the same time, the spectral quality is relatively improved contrast to Fig. 7(c). It seems that the DSVM method preserves the spectral information better in Fig. 7(f). On the other hand, the spatial quality is not ideal. Therefore, we get the fused image that has better spatial quality by choosing the suitable parameter of ζ in Fig. 7(e).

Also we have the objective indices as shown in table 3.

In table 3, the bold font is the best one in the same row. Apparently, the method DSVM behaves best in index Q-average, CC(hyperspectral image) and SAM. Those two indices represent the ability of preserving the spectral information. On the other hand, CC(hyperspectral image) represents the relationship between the original hyperspectral data and fused data. Those three indices demonstrate that the method DSVM does well in preserving the spectral information and this is very important in hyperspectral image fusion. However, the morphology behaves better in AG, it gets a relatively better spatial vision than the others but the worst spectral quality of all. On the other hand, the result of the MSVM is the second best in AG and CC(master image) and it behaves better in SAM than morphology.

The actual spectral curves of pixel(100,100) are displayed in Fig. 8. In all the images, the red curves are from the original data while the blue curves are from the fused data.

The performance of the methods are the same as that in experiment 1. The curves correspond to the objective indices in table 3, respectively. Also, it is obviously that the curve obtained by the DSVM method is the best.

Form the previous discussion, we see that both the methods MSVM and DSVM have their own advantages and disadvantages. It is obviously that the parameters in the MSVM is less than those in the DSVM for the application of morphology in the energy functional. However, from the subjective vision and the objective indices, the DSVM behaves better than the MSVM. Especially in the performance of preserving the spectral information. In a word, DSVM is better than MSVM with proper parameters.

5 Conclusions

In this paper, we propose a new variational model which is based on the similarity measure for hyperspectral image fusion. Its basic idea is to transform the image fusion process into an optimization problem. After the variational model is established on the basis of the two typical similarity measures, we use the Split Bregman Iteration to obtain the extremal function which represents the fused image. Also, we analyze the convergence of the algorithm.

Contrast to the conventional methods, we see the DSVM and the MSVM are more suitable to be used for hyperspectral image fusion because they enhance the low spatial resolution image while preserving the spectral property very well. The multiscale morphology method just does well in image vision and the wavelet method does not perform well in the spectral preservation. The MSVM performs better in reflecting the details of images but relatively worse in the spectral preservation, while the DSVM do well in both two sides, so it is the best of all except for more parameters.

For the practical applications, one important thing that we should consider is the choice of parameters in the models. Most of the parameters in our experiments are obtained manually. Therefore, it is one of the problems we have to solve for the automatical fusion. Besides, both the DSVM and the MSVM are based on the spatial domain. So the discussion based on frequency domain is another possible choice.

Algorithm 1 Proposed processing chain for ITS.

1: Initialization: given an ITS $\mathbf{F}^{I\times J\times K}$, sampling window size $w\times h$, step size s, maximum iteration times $maxiter$, threshold δ, $\mathbf{I}\text{ndex}_{all} = \emptyset$ and matrix set $\mathbf{M}_{all} = \emptyset$ (They are used to respectively store iteration number and the calculated basis matrix in the iteration).

2: Transform the ITS in RGB color module into a new version in N-RGB color module.

3: Implement the image simulating process with the sampling parameters w, h and s. Obtain the IMB $\mathbf{G}^{W\times H\times L}$ where L is the band number and $L = w \times h \times s$.

4: **for** $kk = 1$ to $maxiter$ **do**

5: Randomly initialize the $p \times p$ auxiliary matrix $\mathbf{B}$;

6: **for** $ii = 1$ to p **do**

7: Implementing the equation: $\mathbf{f} = ((\mathbf{I} - \mathbf{B}\mathbf{B}^{+}))/\|(\mathbf{I} - \mathbf{B}\mathbf{B}^{+})\|_F$, we obtain the vector $\mathbf{f}$ that is orthonormal to the subspace spanned by $\mathbf{B}$. $\|\cdot\|_F$ is the Frobenius norm. $\mathbf{B}^{+}$ could be calculated with the function *pinv* in the MATLAB.

8: Project the data $\mathbf{A}_p$ into the vector $\mathbf{f}$ by $\mathbf{v} = \mathbf{f}^T\mathbf{A}_p$.

9: Calculate the extreme point of $\mathbf{v}$ and update the auxiliary matrix $\mathbf{B}$.

$$k = argmax_{j=1,\ldots,N}|\mathbf{v}(j)|. \tag{32}$$

$$\mathbf{B}(:, ii) = \mathbf{A}_p(:, k) \tag{33}$$

10: Update the expected vertex matrix:

11: **end for**

12: Store the indices of simplex and calculate the volume of simplex (Vos) spanned by $\mathbf{M}$ via implementing equation (2)

$$\mathbf{I}\text{ndex}_{all} = \{\mathbf{I}\text{ndex}_{all}, index\} \tag{34}$$

$$\mathbf{M}_{all} = \{\mathbf{M}_{all}, \mathbf{M}\} \tag{35}$$

$$Vos(kk) = V(\mathbf{M}_{all}(kk)) \tag{36}$$

13: **end for**

6 Acknowledgments

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7 Tables

Table 1: Objective indices of the experiment 1

	wavelet	morphology	MSVM	DSVM
Q	0.8007	0.6813	0.5048	**0.9363**
AG	0.0051	0.0057	**0.0058**	0.0050
CC(master image)	**0.8418**	0.8158	0.7859	0.7276
CC(hyperspectral image)	0.8959	0.8695	0.8877	**0.9626**
SAM	10.4858	19.1959	18.0660	**6.5864**

Table 2: The ratios of terms in MSVM and DSVM

	E_g/E_g	E_s/E_g	$E_v^{'}/E_g$
MSVM	1	0.433	1.131
	E_g/E_g	E_s/E_g	$E_v^{''}/E_g$
DSVM	1	2.948	4.293

Table 3: Objective indices of the experiment 2

	wavelet	morphology	MSVM	DSVM
Q	0.7176	0.4300	0.5048	**0.8813**
AG	0.0111	**0.0126**	0.0121	0.0083
CC(master image)	0.8912	**0.9317**	0.9016	0.7647
CC(hyperspectral image)	0.8575	0.7617	0.8096	**0.9517**
SAM	2.9218	8.6977	8.1096	**2.2920**

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obtained by the DSVM method, (f) The fused image with high spectral resolution obtained by the DSVM method.

Fig. 5 Spectral responses of the pixel (100,100) in the experiment 1. (a) The spectral curves obtained by wavelet method. (b) The spectral curves obtained by morphology method. (c) The spectral curves obtained by MSVM method. (d) The spectral curves obtained by DSVM method.

Fig. 6 Original images for experiment 2. (a) Band 4 of hyperspectral image. (b) The master image.

Fig. 7 Original false color image and fused false color images obtained by using different methods in experiment 2. (a) The original hyperspectral image. (b) The fused image obtained by the wavelet method, (c) The fused image obtained by the multiscale morphology method, (d) The fused image obtained by the MSVM method, (e) The fused image with high spatial resolution obtained by the DSVM method, (f) The fused image with high spectral resolution obtained by the DSVM method.

Fig. 8 Spectral response of pixel (100,100) in the experiment 2. (a) The spectral curves obtained by wavelet method. (b) The spectral curves obtained by morphology method. (c) The spectral curves obtained by MSVM method. (d) The spectral curves obtained by DSVM method.

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